

Pheromone output of a simple olfactometer

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1 Introduction

This short treatise is an attempt to calculate the molecule output of a simple olfactometer that is based on placing a filter paper with adsorbed odorant molecules into a tube with a laminar constant airstream. For describing the system we are making some simplifying assumptions as detailed in the corresponding sections.

Direct measurements have suggested that the molecule output should be proportional to the total number of molecules deposited and to the speed of the airflow. With our assumptions and in our ordinary differential equation (ODE) description of the system we find the first rule but by no means the second.

2 Point source model without re-adsorption

In the simplest description we could make the following assumptions:

- There is a constant rate α of adsorption and constant rate β of desorption.
- The pheromone was deposited in a point without any spatial spread.
- The flow in the olfactometer is perfectly laminar and has constant speed v .
- The adsorption rate is so small compared to the desorption rate that it can be neglected.

Using these assumptions we can formulate a simple ODE for the total number of desorbed molecules N_β :

$$\frac{dN_\beta}{dt} = \beta N_\alpha = \beta(N_0 - N_\beta) \quad (1)$$

where $N_\alpha = N_0 - N_\beta$ is the number of still adsorbed molecules and N_0 is the original number of molecules present. Accordingly the boundary condition is $N_\alpha = N_0$ and $N_\beta = 0$. The ODE (1) is easily solved to yield

$$N_\beta(t) = N_0(1 - \exp(-\beta t)) \quad (2)$$

which then gives a rate of molecules coming out of the end of the olfactometer as

$$n_{\text{out}}(t) = \beta N_0 \exp(\beta(t - l/v)) \quad (3)$$

where l is the length of the olfactometer and v the speed of flow in it (see figure 1).

In this description, clearly, the rate of pheromone molecules leaving the olfactometer is described by an exponential function decaying with rate β . The initial output rate is proportional to the total number of deposited molecules N_0 and the desorption rate β . The speed of flow does not play a role. Intuitively this is justified by the assumption that simply all molecules that desorbed at some time t will appear l/v time later at the end of the olfactometer. The delay depends on v but the number of molecules delivered per time not.

3 Volume source with re-adsorption

In a somewhat more realistic model we may assume that pheromones are released in some volume $l_r A$ and there is a non-negligible re-adsorption rate. In this case the ODE governing the number of airborne pheromone molecules N_β which are present in the release volume (note the change of meaning of N_β) is

$$\frac{dN_\alpha}{dt} = \alpha N_\beta - \beta N_\alpha. \quad (4)$$

$$\frac{dN_\beta}{dt} = \beta N_\alpha - \alpha N_\beta - N_\beta v/l_r \quad (5)$$

where N_α is the number of adsorbed molecules. This 2d system of first order autonomous ODE can also be solved analytically with solution

$$N_\alpha(t) = k_1 \exp(\lambda_+ t) + k_2 \exp(\lambda_- t) \quad (6)$$

$$N_\beta(t) = \frac{\beta}{\lambda_+ + \alpha + v/l_r} k_1 \exp(\lambda_+ t) + \frac{\beta}{\lambda_- + \alpha + v/l_r} k_2 \exp(\lambda_- t) \quad (7)$$

$$\lambda_\pm = -\frac{\beta + \alpha + v/l_r}{2} \pm \Gamma \quad (8)$$

$$\Gamma = \sqrt{\left(\frac{\beta + \alpha + v/l_r}{2}\right)^2 - \frac{\beta v}{l_r}} \quad (9)$$

$$k_1 = N_0 - k_2 \quad (10)$$

$$k_2 = \frac{N_0 \alpha \beta}{(\alpha + v/l_r - \beta)\Gamma + 2\left(\frac{\beta - \alpha - v/l_r}{2}\right)^2 + 2\alpha\beta}. \quad (11)$$

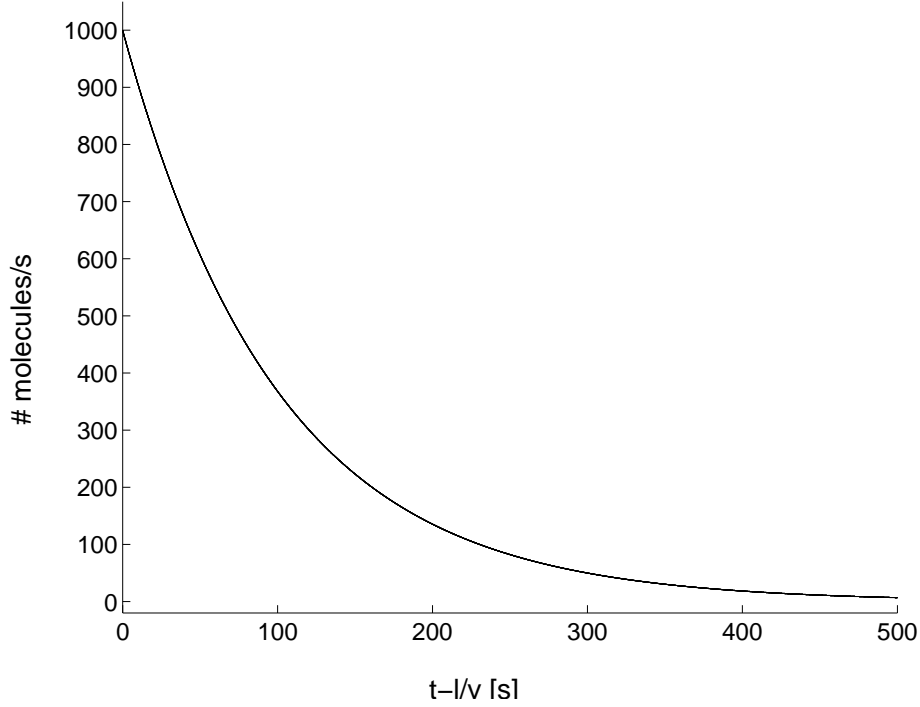


Figure 1: Number of molecules per second exiting the olfactometer with the simple point source model. The parameters used to generate this graph were $\alpha = 0/s$, $\beta = 0.01/s$, $l_r = 1cm$ and $N_0 = 10^5$ molecules.

Clearly, the number of molecules exiting at time t , $n_{out}(t) = v/l_r N_\beta(t - l/v)$ is still proportional to N_0 which is quite trivial. The dependence on v is almost impossible to see by naked eye. Figure 2 shows some examples for this. Interestingly the trend changes halfway through the process: Initially, the higher flow rates give more output but as the source depletes, the trend inverts.

Using higher concentrations, i.e., more molecules initially, does not make any difference in this framework as the adsorption and desorption rates were formulated per molecule. I am personally not really sure whether this is a good description but if the molecules do not interact too much either in adsorption or in the air I would think it is reasonable.

One extension one could think about is that, maybe, the filter paper is placed and the desorption/adsorption takes place for a while when the airflow is still off. The the airflow is switched on after the airborne and adsorbed molecules are already in a steady state. I could easily redo the calculation for that situation.

Also, it would be helpful to be able to put realistic numbers for the different quantities. Maybe I am orders of magnitude off with the adsorption and desorption constants and/or the flow rate.

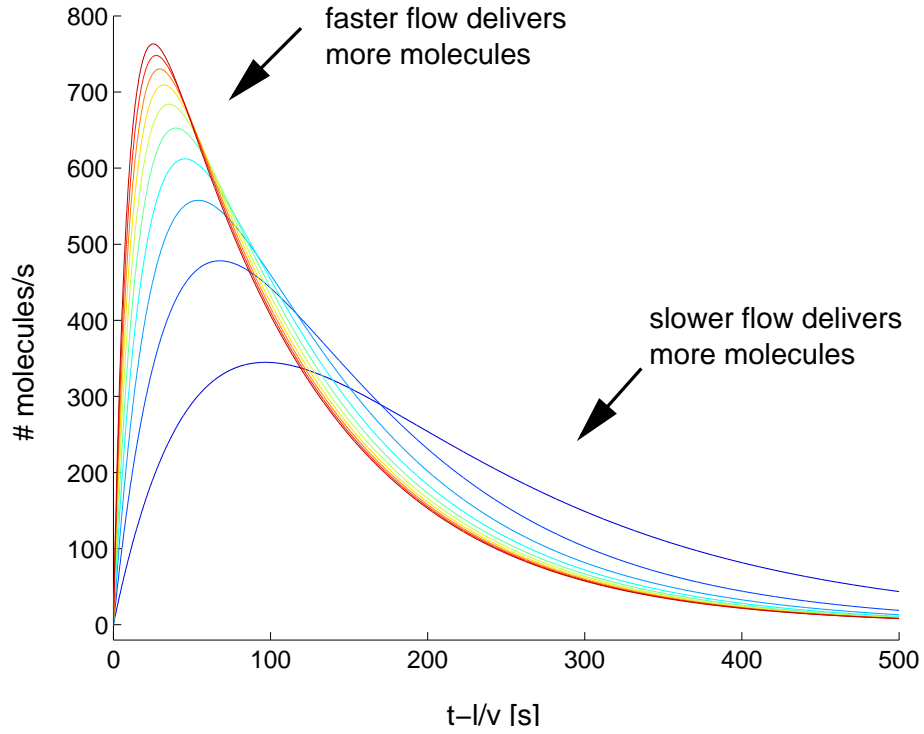


Figure 2: Number of molecules per second exiting the olfactometer. The colours are for $v = 0.0001\text{cm/s} \dots 0.001\text{cm/s}$ in steps of 0.0001cm/s (from cold to hot colours). The other parameters are $\alpha = 0.002/s$, $\beta = 0.01/s$, $l_r = 1\text{cm}$ and $N_0 = 10^5$ molecules.

4 Conclusions

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