
Biophysical models of sensillum and action potential generator



Pherosys meeting 22 June 2009

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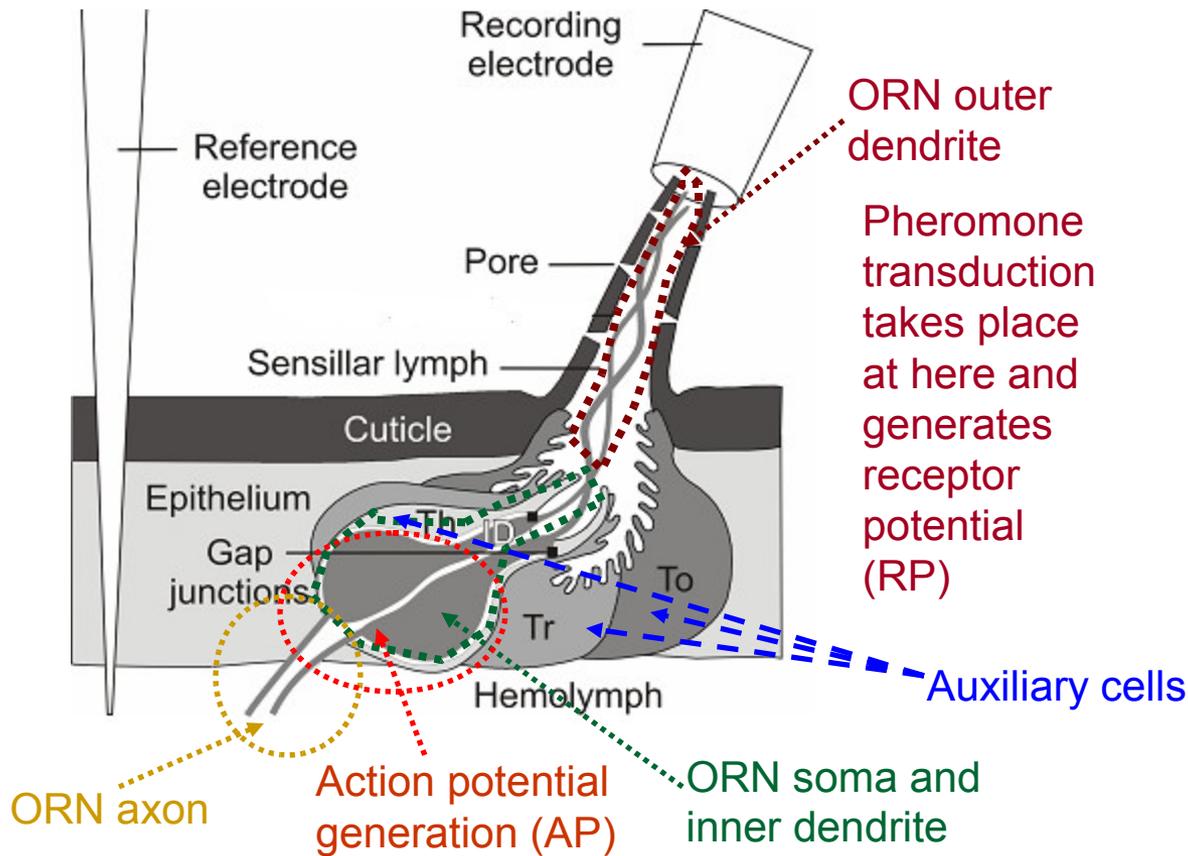
INRA , UMR 1272 PISC

Experimental background

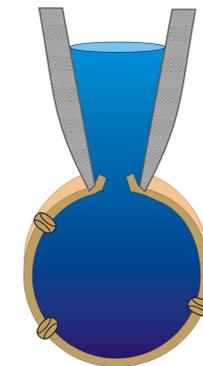
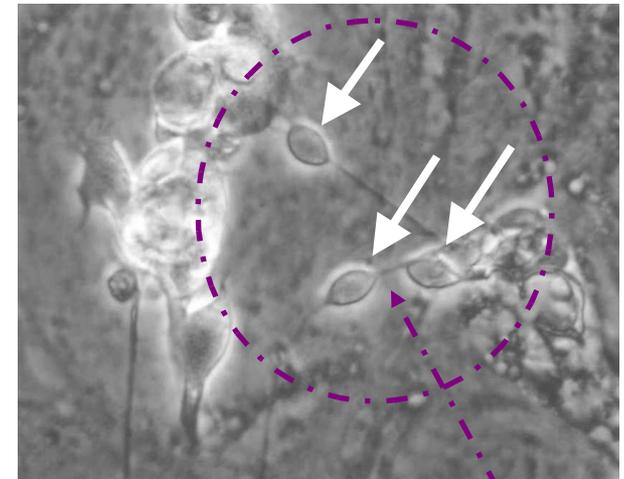


- Moth pheromone-sensitive sensillum trichodeum in tip-recording conditions

Record the extracellular potential SP at the tip which is related to the RP



- Primary culture of ORNs and whole-cell patch-clamp



Three ORN cells

Aims

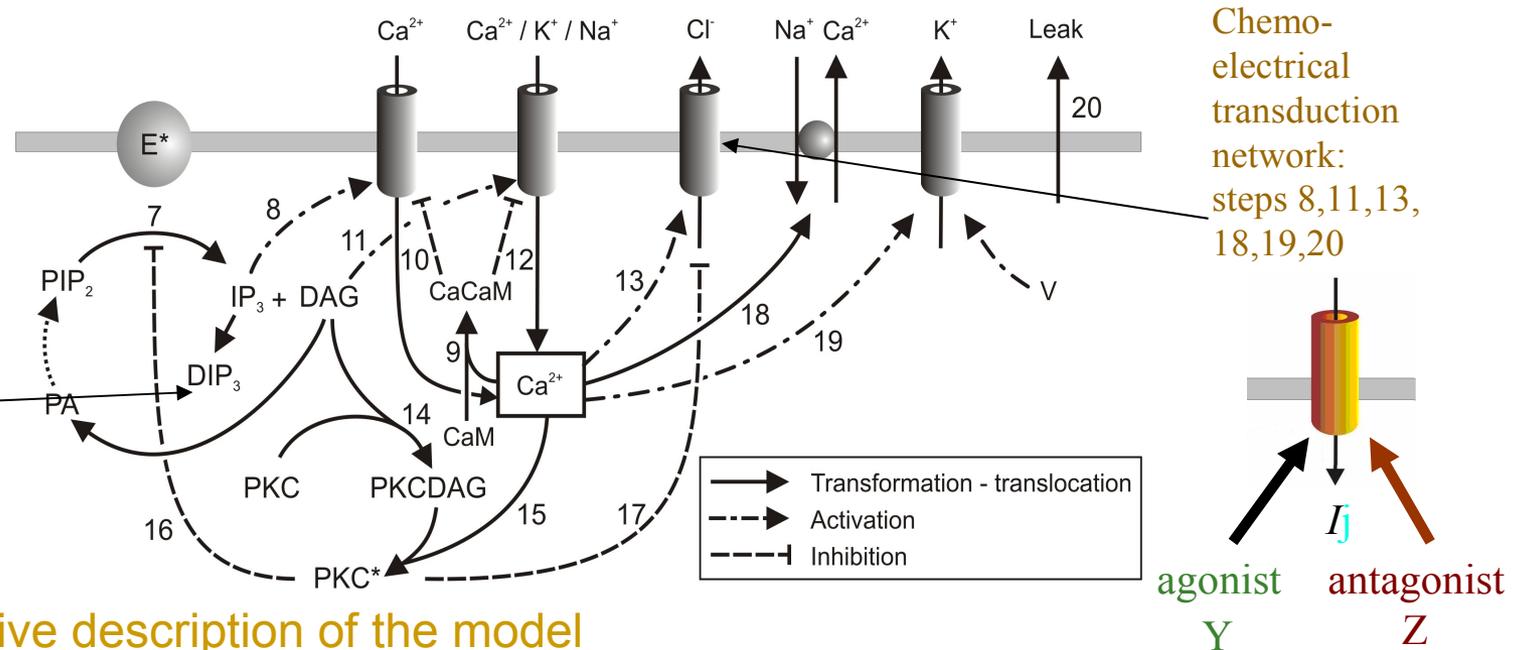


- I. Mathematically describe the chemo-electrical transduction taking place at the sensillar lymph and ORN outer dendrite.
- II. Using compartmental model to investigate the sensillar circuitry and quantitatively fit to the dose response curves obtained by *tip-recording experiments*.
- III. Employing Hodgkin-Huxley type model to study the ionic channels generating action potential at the ORN soma and fit to the data obtained by *patch-clamp experiments*.



I. Mathematically describe the chemo-electrical transduction taking place at the ORN outer dendrite

➤ The qualitative model



Biochemical reaction Network: steps 7,9, 10,12,14, 15,16,17

Chemo-electrical transduction network: steps 8,11,13, 18,19,20

➤ The quantitative description of the model

Each reaction is described by mass action law $A + B \xrightleftharpoons[k-1]{k1} C$ $\frac{d[C]}{dt} = -\frac{d[A]}{dt} = -\frac{d[B]}{dt} = k_1[A][B] - k_{-1}[C]$

The synthesis rate of the second messenger IP₃ and DAG is described by $v = \frac{S_{M1}}{1 + (PKC^*(t) / K_{is})^{n_{is}}} \cdot E^*(t)$

Each ionic current, I_j, is given by Ohm's law $I_j = G_j(V_{ed} - V_{id} + E_j)$

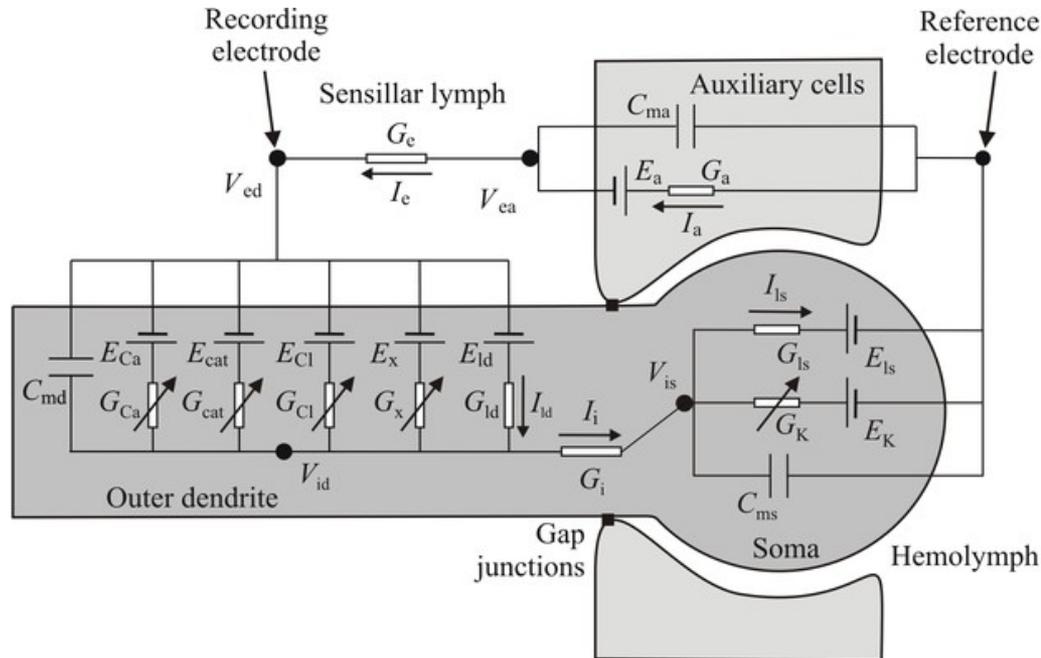
$$K_j = K_{mj} \left(1 + \frac{i_{Mj} - 1}{1 + (K_{ij}/Z)^{n_{ij}}} \right) \quad G_j = \frac{G_{Mj}}{1 + (K_j/Y)^{n_j}}$$

concentration of the antagonist $\xrightarrow{\quad}$ $\xrightarrow{\quad}$ $\xrightarrow{\quad}$ concentration of the agonist

Modeling the electrical network at the sensillum



➤ The equivalent circuit of the three-compartment model



➤ The quantitative description of the electrical network

The mathematical description of each compartment, k , can be derived from *Kirchhoff's current law*

$$I_{mk} = C_{mk} \frac{dV_k}{dt} + I_{ionk}$$

The receptor potential (RP) at the dendrite is

$$RP = (V_{id} - V_{ed}) - (V_{id}^0 - V_{ed}^0)$$

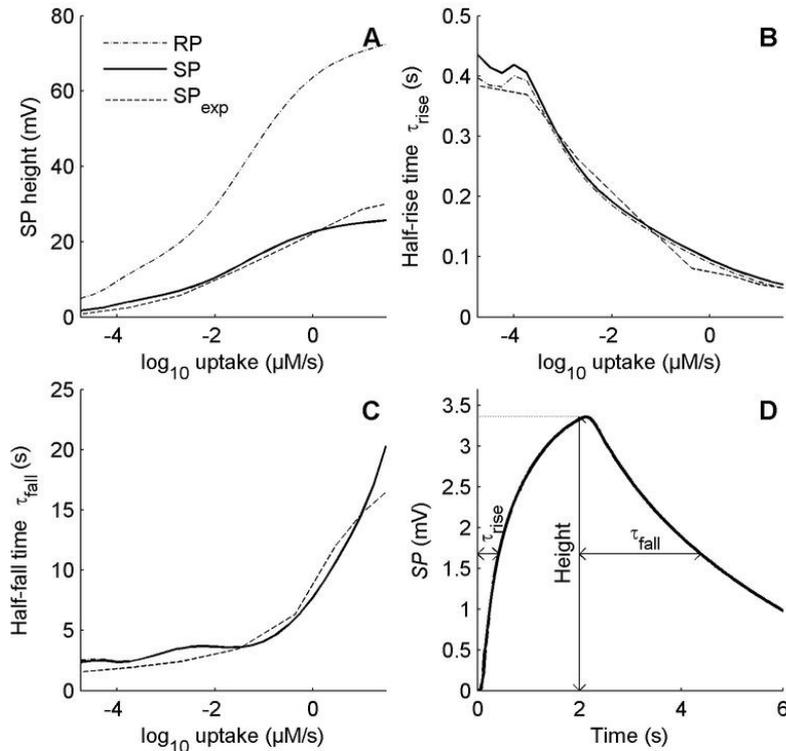
The measured sensillum potential (SP) by tip-recording experiment is

$$SP = V_{ed} - V_{ed}^0$$

Fitting to the tip-recorded dose response characteristic



function by three-compartment model

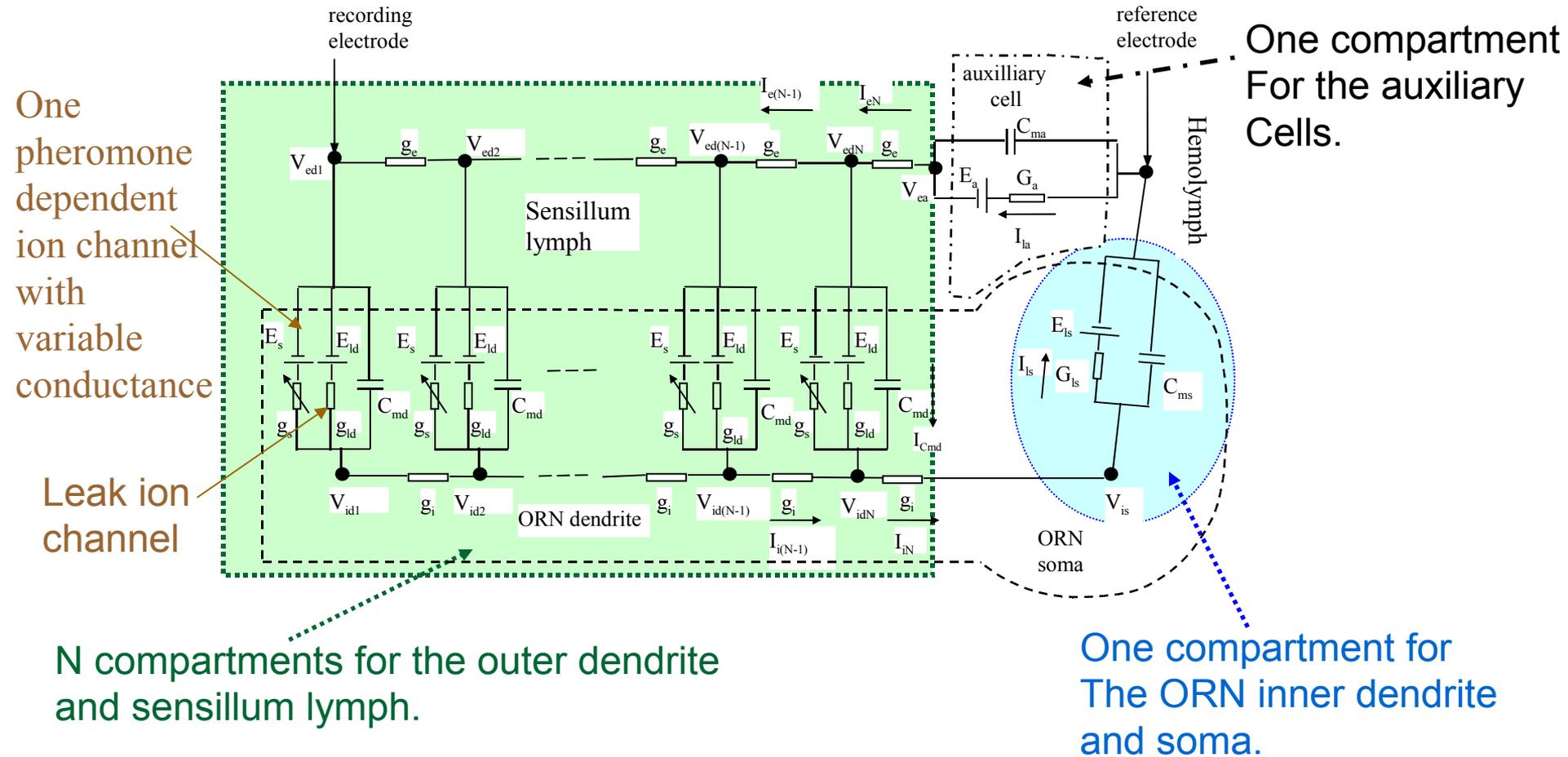


- The heights of *SP* & *RP* are very different, by contrast, their half-rise and half-fall times are closer.
- The model obtained captures the main features of the dose-response curves of SP:
 - 1) the wide dynamic range from $10^{-4.75} \mu\text{M/s}$ to $10^{1.5} \mu\text{M/s}$ of 6 decades with almost the same saturation amplitudes as the experimental data;
 - 2) the short rising time;
 - 3) the long falling time.

Sensillum model with multicompartmental outer dendrite and its simplification



➤ The circuit of pure electrical sensillar model with N+2 compartments



➤ The quantitative description of the N+2-compartment model



Differential equation
for the potential inside
the ORN soma

$$\frac{dV_{is}}{dt} = (I_{iN} - I_{Gls}) / C_{ms}$$

Differential equation
for the potential outside
the auxiliary cells

$$\frac{dV_{ea}}{dt} = (I_{la} - I_{eN}) / C_{ma}$$

Differential equation
for the potential inside
the jth compartment
of ORN outer dendrite

$$\begin{aligned} \frac{dV_{idj}}{dt} = & \frac{g_e}{C_{md}(g_e + g_i)} (I_{ej-1} - I_{ej} + I_{ldj} + I_{sj}) \\ & + \frac{g_e}{C_{ma}(g_e + g_i)} (I_{la} - I_{eN}) + \frac{g_i}{C_{ms}(g_e + g_i)} (I_{iN} - I_{ls}) \end{aligned}$$

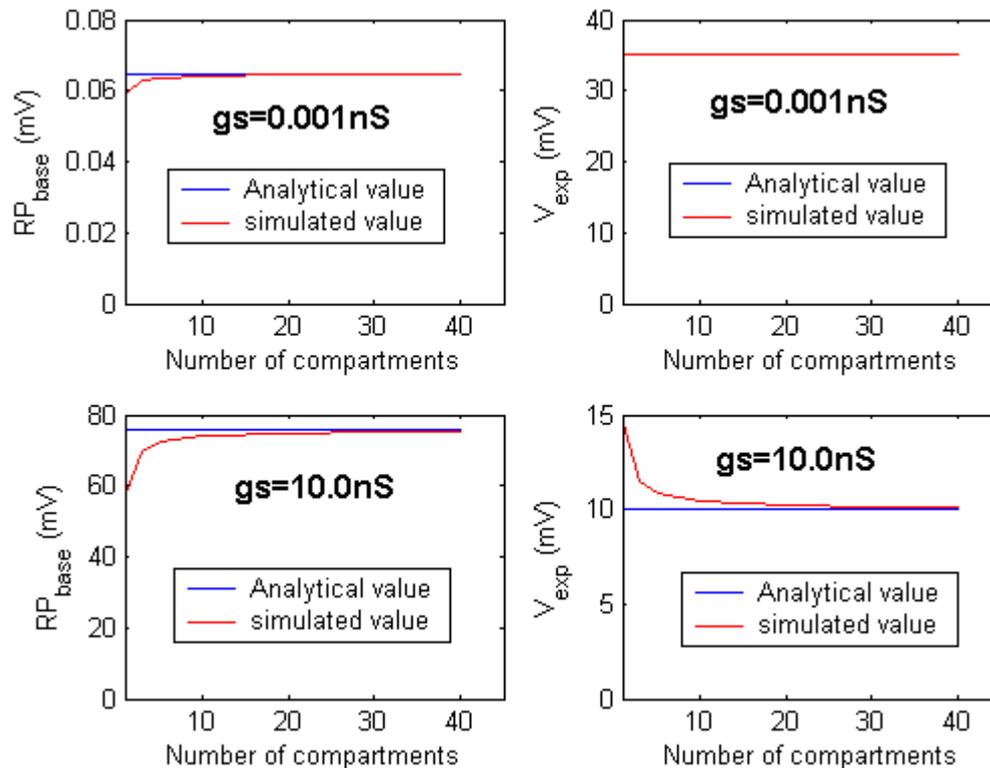
Differential equation
for the potential outside
the jth compartment
of ORN outer dendrite

$$\begin{aligned} \frac{dV_{edj}}{dt} = & \frac{g_i}{C_{md}(g_e + g_i)} (I_{ej} - I_{ej-1} - I_{ldj} - I_{sj}) \\ & + \frac{g_e}{C_{ma}(g_e + g_i)} (I_{la} - I_{eN}) + \frac{g_i}{C_{ms}(g_e + g_i)} (I_{iN} - I_{ls}) \end{aligned}$$

Comparing the steady state values of the model with the analysis results



Computer simulations for models with 1 to 40 dendrite compartments were performed and the resulting steady state values were compared with their analytical value obtained by Vermeulen and Rospars in *Eur Biophys J* (2001) 29:587-596.



The relative error increases with pheromone dependent conductance g_s and decreases with number of compartments. It is less than 1% with 40 dendrite compartments and greater than 24% for $g_s = 10 \text{ nS}$ with 1 dendrite compartment.

Effect of the battery E_a situated at the auxiliary cells

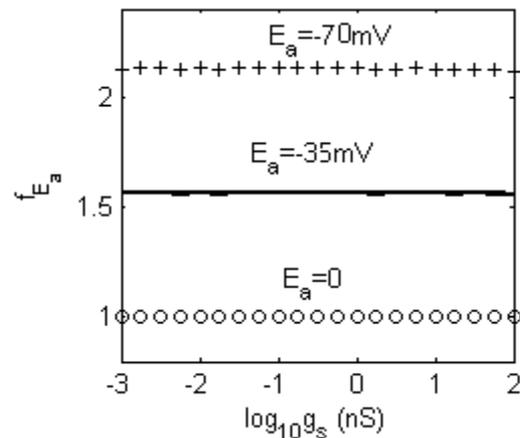


Experimental hypothesis: E_a plays a functional role in increasing sensitivity.

The height of RP at the dendrite base

Amplification factor of E_a $f_{E_a}(\log g_s) = \frac{H_{RP_s}(\log g_s, E_a)}{H_{RP_s}(\log g_s, E_a = 0)}$

Pheromone dependent conductance

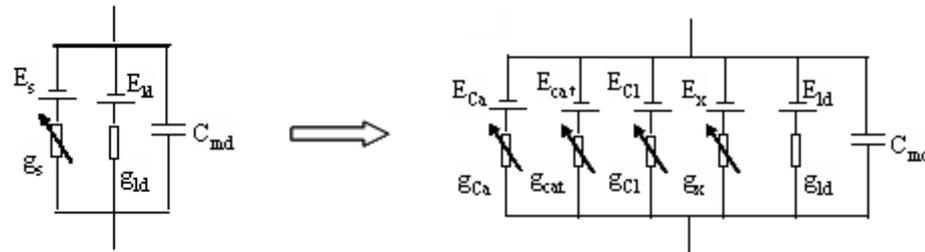


The effect of E_a is linear amplification. The amplification proportion for different stimulation dependent g_s is the same.

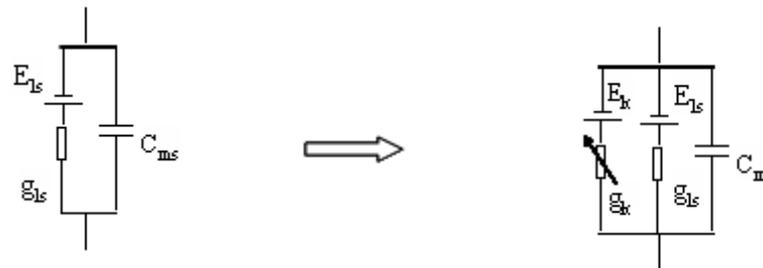
Multicompartmental sensillar model with pheromone transduction cascade and realistic channels



The circuit of each compartment at the ORN outer dendrite

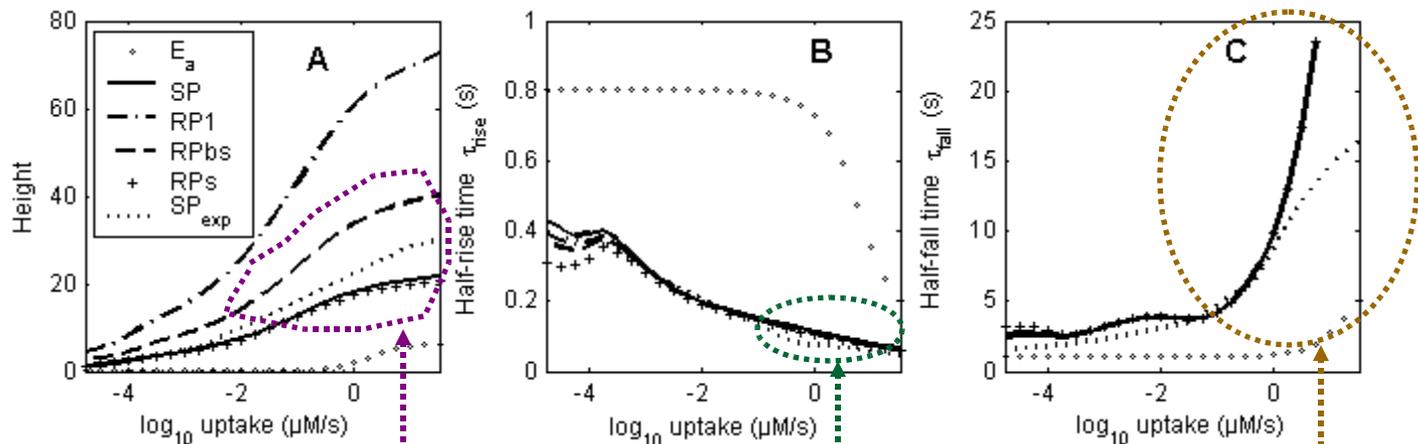


The circuit of the ORN soma

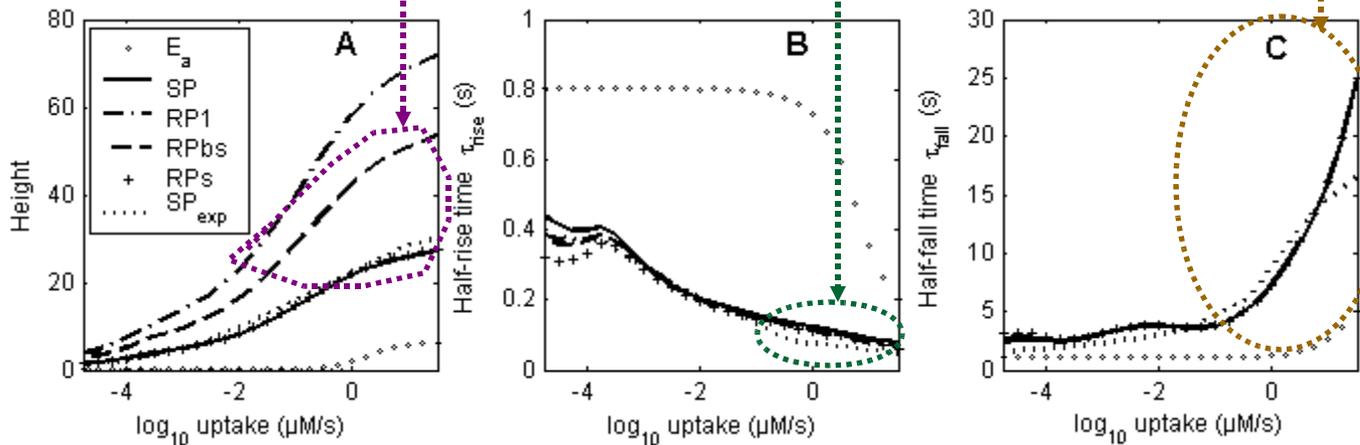




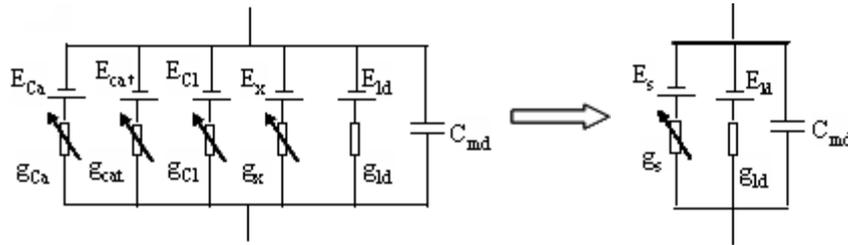
Result with one compartment at the outer dendrite



Result with 20 compartments at the outer dendrite



Simplification of the multichannel model



Mathematical conversion:

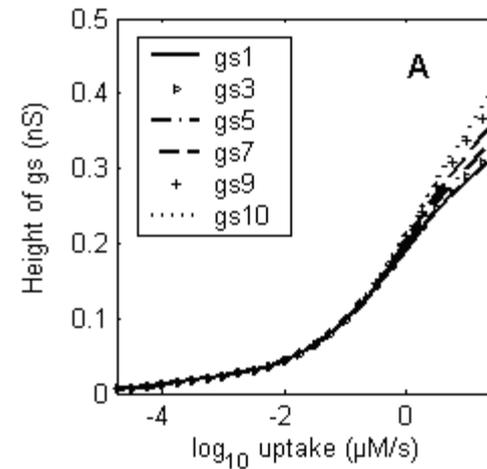
$$I_{sj} = I_{Caj} + I_{ctj} + I_{Clj} + I_{xj}$$

$$I_{sj} = g_{sj}(V_{edj} - V_{idj} + E_s)$$

$$g_{sj} = (I_{Caj} + I_{ctj} + I_{Clj} + I_{xj}) / (V_{edj} - V_{idj} + E_s)$$

For simplicity, let $E_s = 0$, using

this equation, we get g_{sj} for each compartment based on the simulation result on the multiple channels and multiple compartments model.



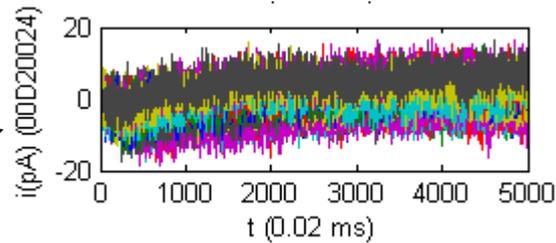
III. Modeling the ion currents through voltage-gated channels generating action potential at the soma and axon initial segment of ORN



The raw data by Philippe Lucas

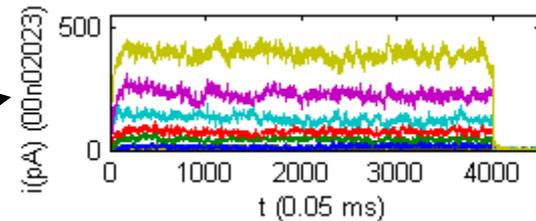
Fast sodium current

Transient Ca²⁺ current

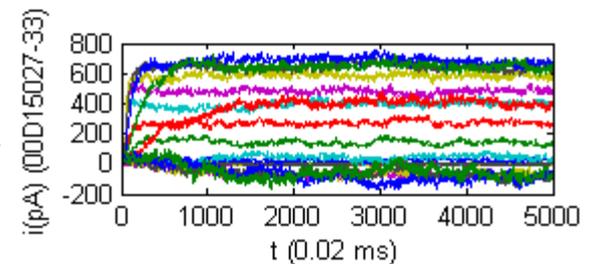


Delayed rectifier potassium current

Transient potassium A current



Calcium-dependent potassium current



III Data preprocessing



Step 1: Removing the artifacts

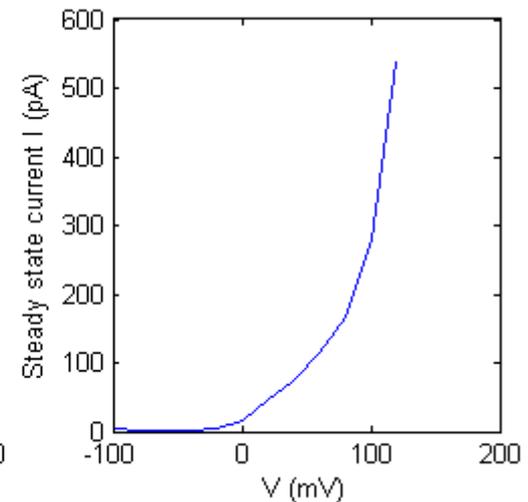
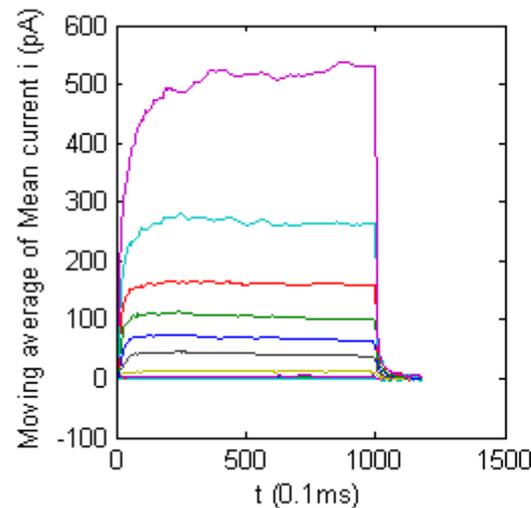
- 1) Taking the average of data obtained from 10 to 12 experiments.
- 2) Taking the average of the data before and after the artifacts as the value during the artifacts.

Step 2: Removing the noise and smoothing the current curves

Moving average approach

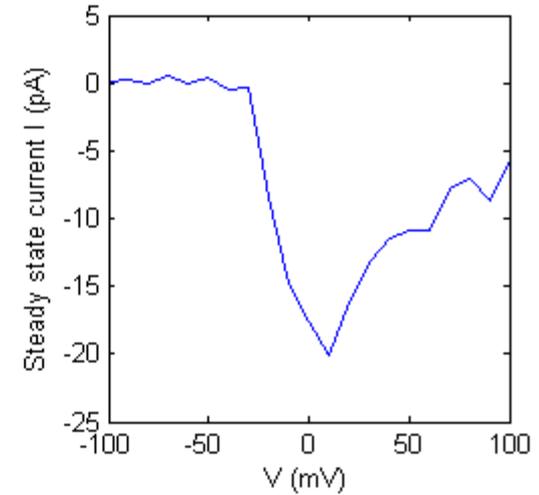
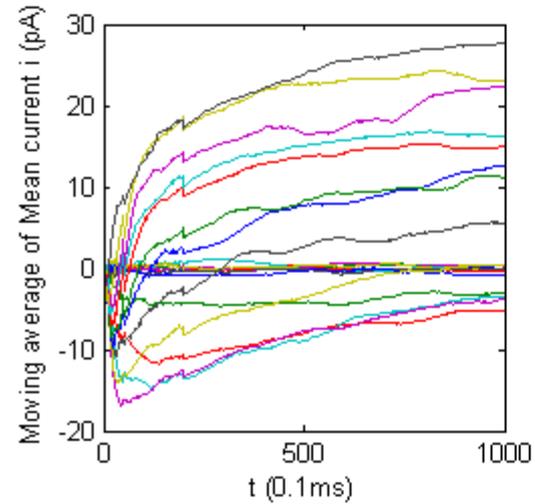
$$y(i) = \frac{1}{M} \sum_{j=0}^{M-1} x(i+j)$$

Processed voltage-dependent delayed rectifier K currents

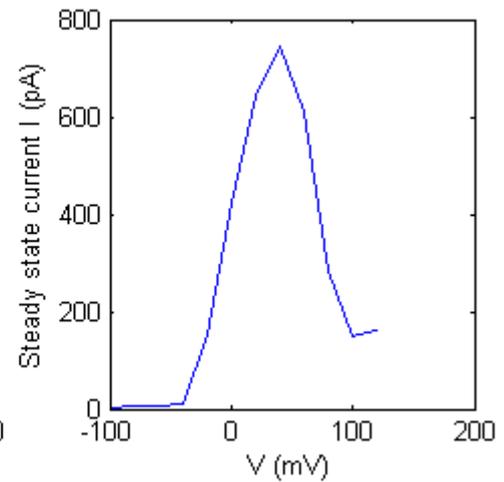
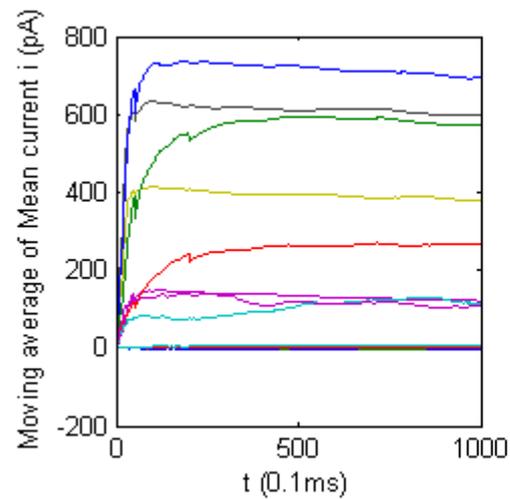




Processed voltage-dependent Ca currents



Processed Ca^v - and voltage-dependent K currents



III Modeling the ion currents by Hodgkin-Huxley type equations



Maximal conductance

Reversal potential

$$I_{ionaj} = g_j m^M h^N (V_{is} - E_j)$$

Gating variables

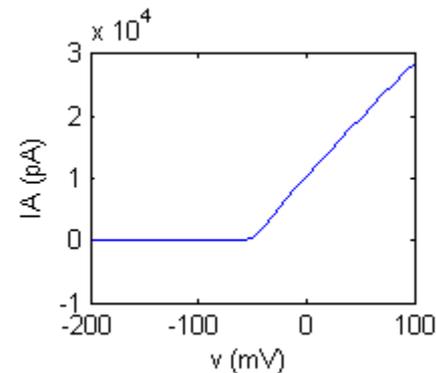
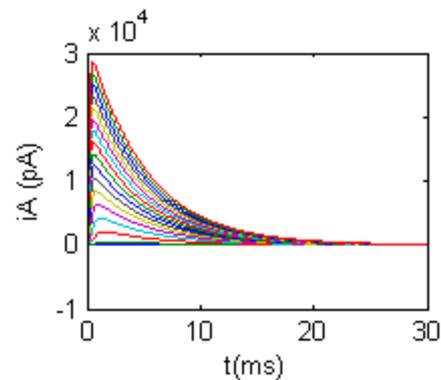
$$\dot{m} = (m_{\infty}(V_{is}) - m) / \tau_m(V_{is}), \quad 0 \leq m(t), h(t) \leq 1$$
$$\dot{h} = (h_{\infty}(V_{is}) - h) / \tau_h(V_{is})$$

Voltage-dependent steady state value

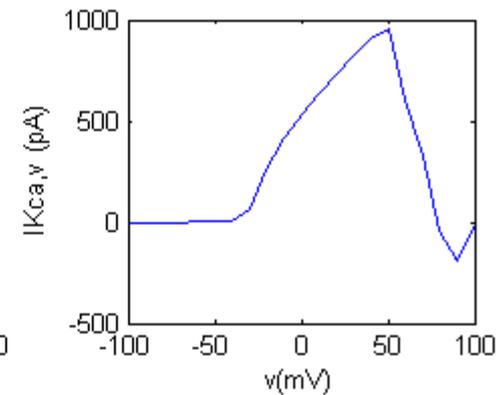
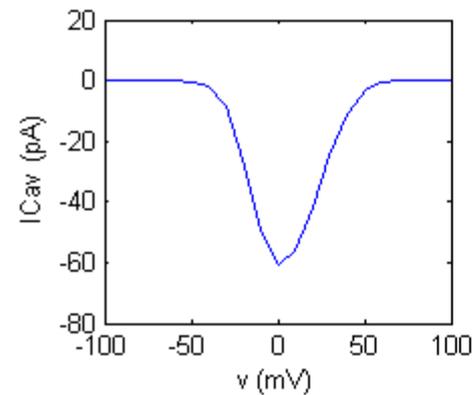
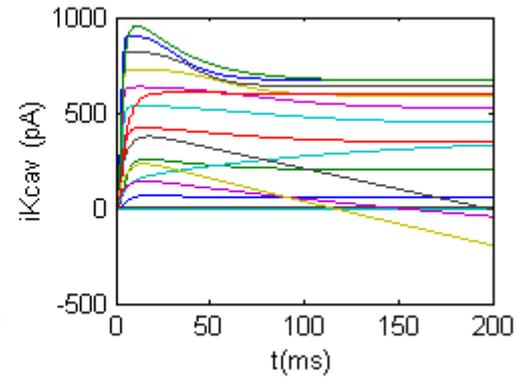
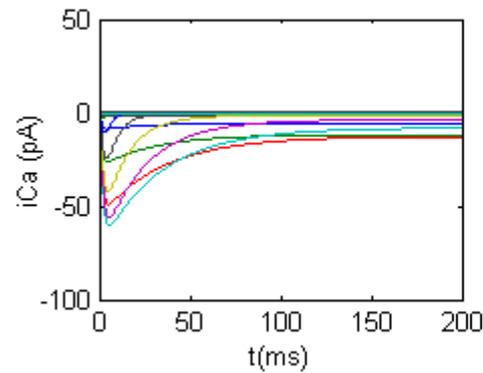
Voltage-dependent time constants

Simulated model results for different type of ion currents

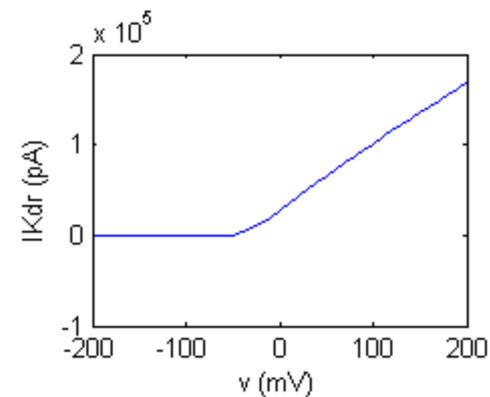
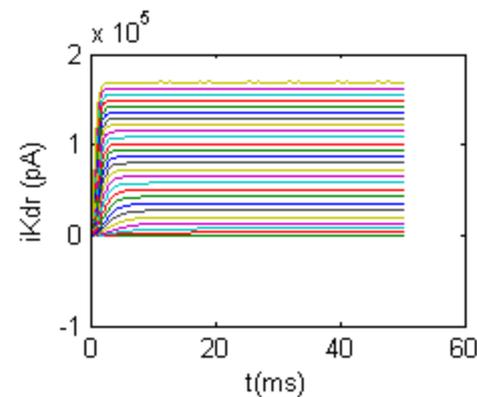
Transient potassium A current



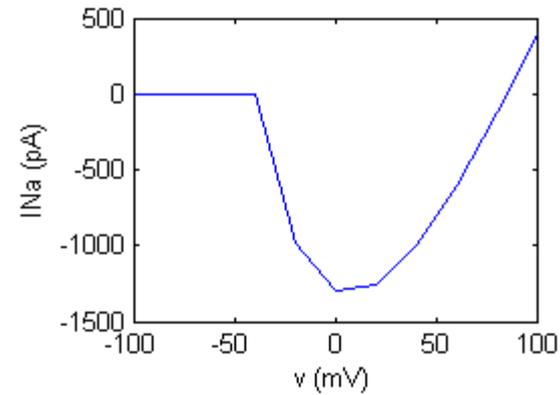
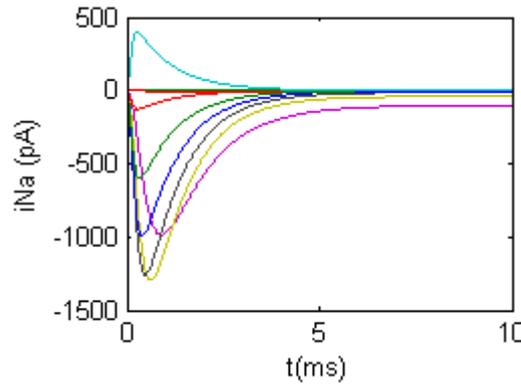
Voltage-gated
Ca currents
and Ca²⁺- and
voltage-dependent
K currents



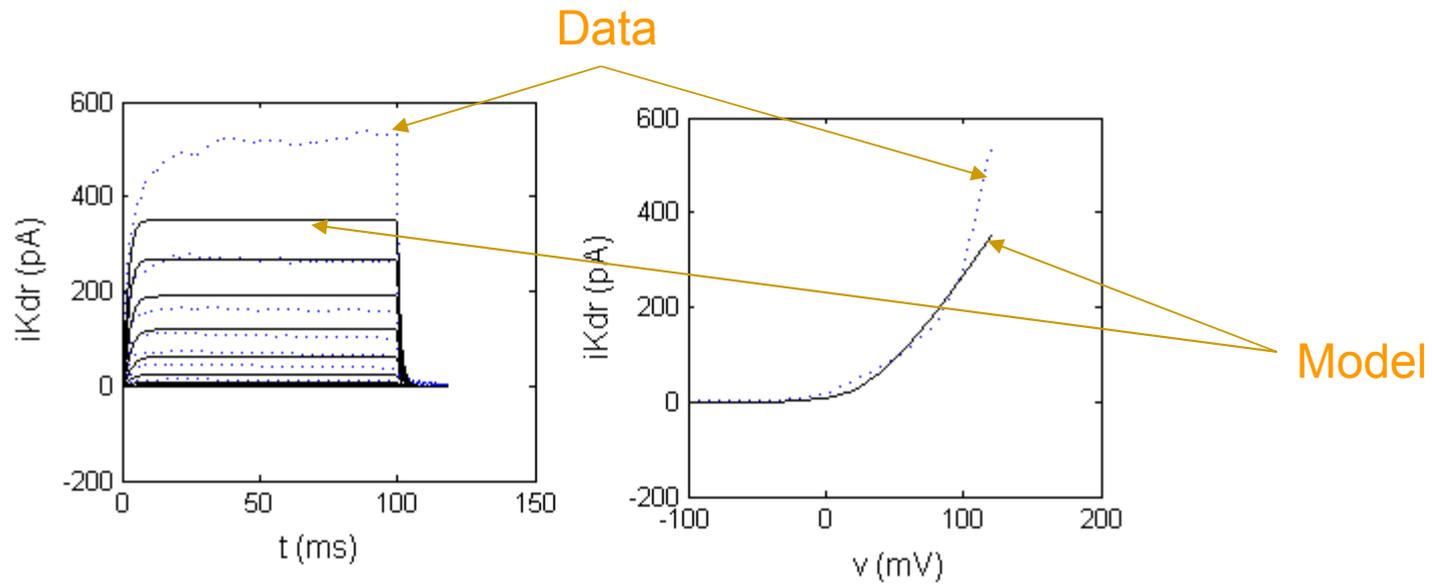
Voltage-gated
delayed rectifier
K currents



Voltage-gated Na currents



Fitting to the data by simulated annealing algorithm



Perspectives



- Properly simplify the detailed sensillar model by including only the major transduction steps and ionic channels: translocation, receptor activation, effector activation, DAG- and Ca^{2+} -gated channels.
- Fitting to the experimental data of the voltage-gated and Ca^{2+} -gated currents at the soma and axon initial segment.
- Built a complete ORN and sensillar model by including the action potential generators — the voltage- and Ca^{2+} -gated

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Thank you for your attention