RiskTorrent: Using Portfolio Optimisation for Media Streaming

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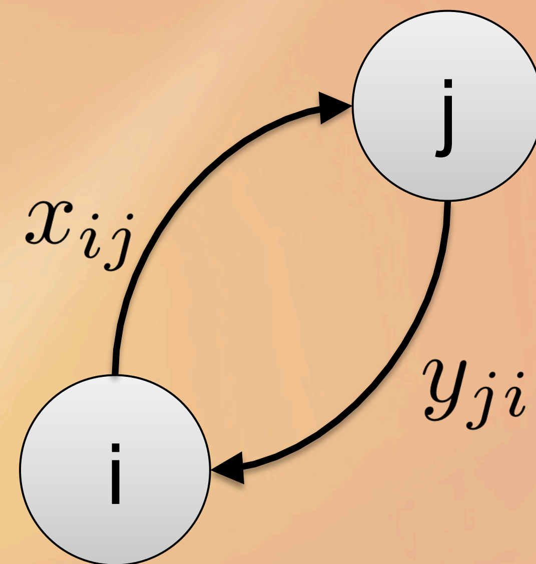
Definitions

- **Reciprocity**: Peers need to upload in order to obtain download capacity

- Let's call x_{ij} the throughput that peer j uploads to peer i

- The throughput that j obtains from i as a result is defined as y_{ji}



Modeling Reciprocity

- The simplest model for $y_{ji} = f(x_{ij})$ is, simply

$$y_{ji} = r_{ij} x_{ij}$$

where $r_{ij} \in \mathbb{R}_+$ can be thought of the return that peer i receives from j, given an investment of x_{ij}

Modeling Total Download Throughput

- The total return (throughput) for peer i is then:

$$Y_i = \sum_{j \in \mathcal{N}} y_{ji} = \sum_{j \in \mathcal{N}} r_{ij} x_{ij}$$

- Thus, in this model, the total return that a peer obtains is a linear combination of the throughput that it allocates to all other nodes

Modeling Download Throughput Variability

- We treat the asset returns r_{ij} as random variables – returns have nonzero ***volatility***
- The variance of Y_i, a linear combination of random variables, is then given by

$$\sigma_{Y_i}^2 = \overline{x}_i^T \Sigma \overline{x}_i$$

where Σ is the covariance matrix of asset returns, and $\bar{x}_i$ is the vector of assigned uploads

Media Streaming: The Investment View

- Each possible allocation of upload bandwidth to specific peers then becomes a ***portfolio***

- For media streaming, we are interested in *minimising throughput variability while maintaining a given stream rate*

- In this case, swarming protocol design becomes ***portfolio selection***

[Markowitz, 1952] and [Markowitz, 1959]

Media Streaming: The Investment View

- The objective is to ***minimise portfolio risk*** while achieving a ***given return*** and satisfying a ***budget constraint***. Diversification helps reduce risk while maintaining returns – the ***volatility*** of the portfolio is smaller than that of its components. Formally:

$$\text{Minimise:} \quad \sigma^2_{Y_i} = \overline{x}_i^T \Sigma \overline{x}_i$$

$$\text{Subject to:} \quad \overline{r}_i^T \overline{x}_i = R_s$$

$$\overline{e}_i^T \overline{x}_i \leq U$$

Media Streaming: The Investment View

- The objective is to ***minimise throughput variability*** while achieving a given ***stream rate*** and satisfying a ***maximum upload capacity constraint***. Formally:

$$\text{Minimise:} \quad \sigma^2_{Y_i} = \overline{x}_i^T \Sigma \overline{x}_i \quad \longleftarrow \text{Throughput Variability}$$

$$\text{Subject to:} \quad \overline{r}_i^T \overline{x}_i = R_s \quad \longleftarrow \text{Constant Stream Rate}$$

$$\overline{e}_i^T \overline{x}_i \leq U \quad \longleftarrow \text{Maximum Upload Capacity}$$

Media Streaming: The Investment View

Minimise: $$\sigma^2_{Y_i} = \overline{x}_i^T \Sigma \overline{x}_i$$ ← Throughput Variability

Subject to: $$\overline{r}_i^T \overline{x}_i = R_s$$ ← Constant Stream Rate

$$\overline{e}_i^T \overline{x}_i \leq U$$ ← Maximum Upload Capacity

$$\overline{x}_i \geq 0$$ ← Non-negativity Constraints (no short-selling)

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What happens if the problem is unfeasible?

Media Streaming: The Investment View

- Usually, this means that the peer has insufficient upload capacity (***capital***) to sustain the required stream rate (***return***)
- In this case, peers fall back to **maximising throughput**, irrespective of risk:

$$\begin{aligned} \text{Maximise:} \quad & \overline{r}_i^T \overline{x}_i \\ \text{Subject to:} \quad & \overline{e}_i^T \overline{x}_i \leq U \\ & \overline{x}_i \geq 0 \end{aligned}$$

Media Streaming: The Investment View

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$$\text{Maximise:} \quad \overline{r}_i^T \overline{x}_i$$

← Stream Rate

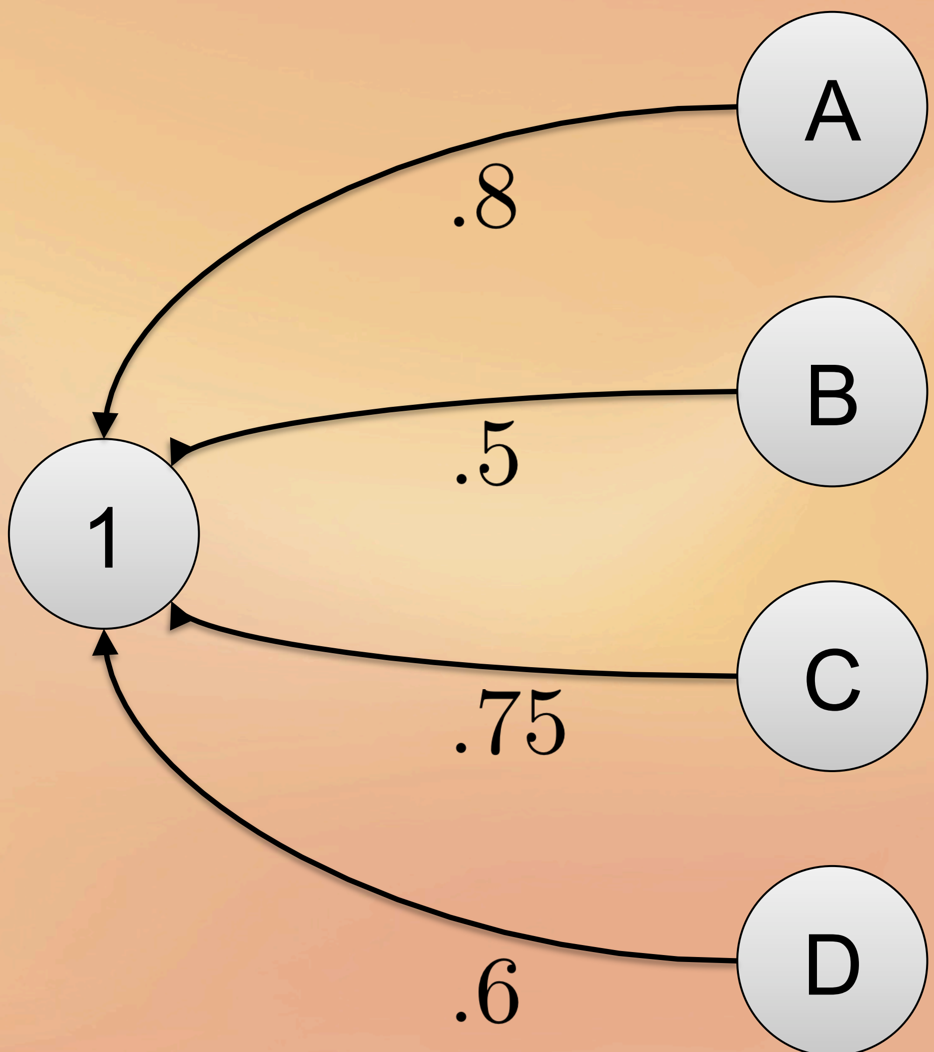
$$\text{Subject to:} \quad \overline{e}_i^T \overline{x}_i \leq U$$

← Maximum Upload Capacity

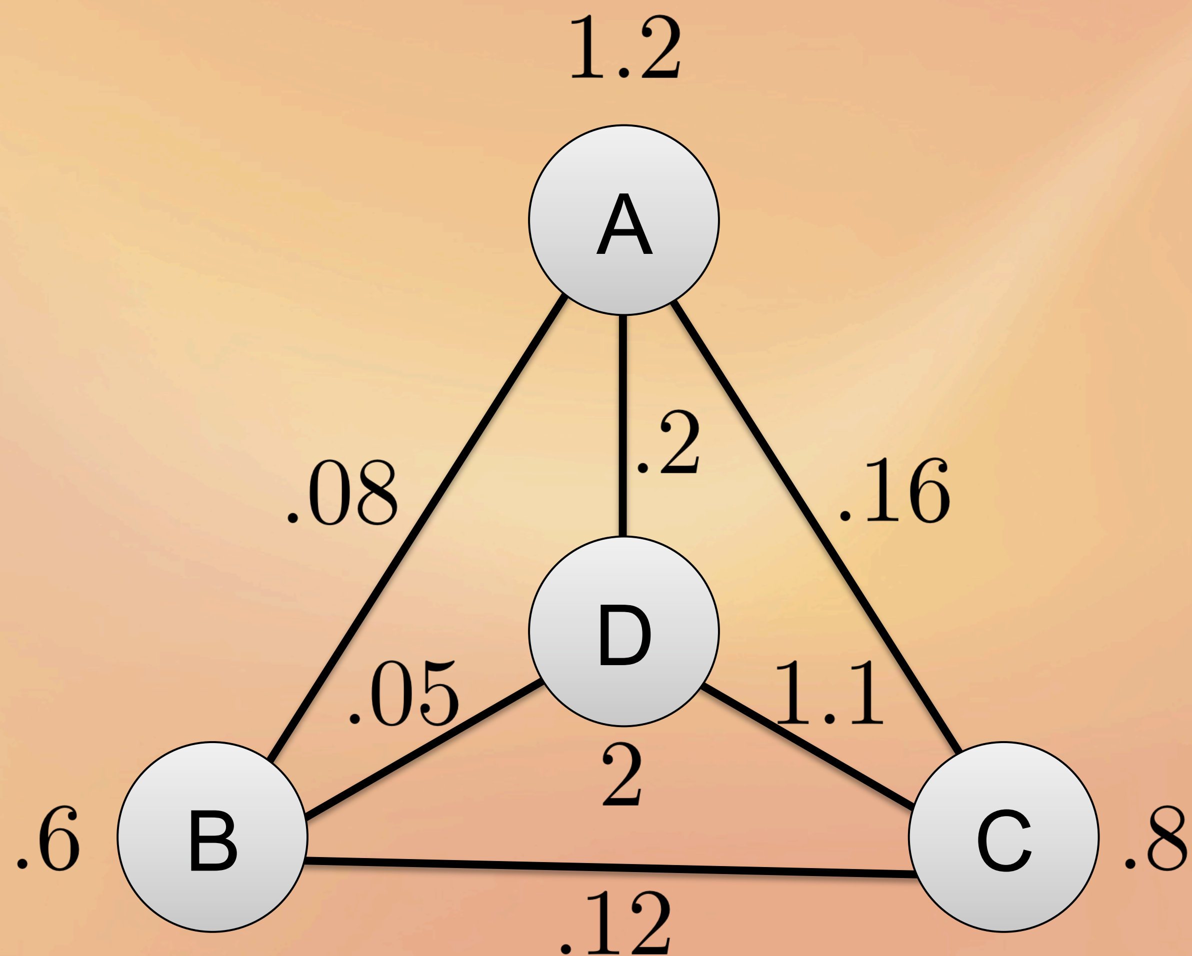
$$\overline{x}_i \geq 0$$

← Non-negativity Constraints (no short-selling)

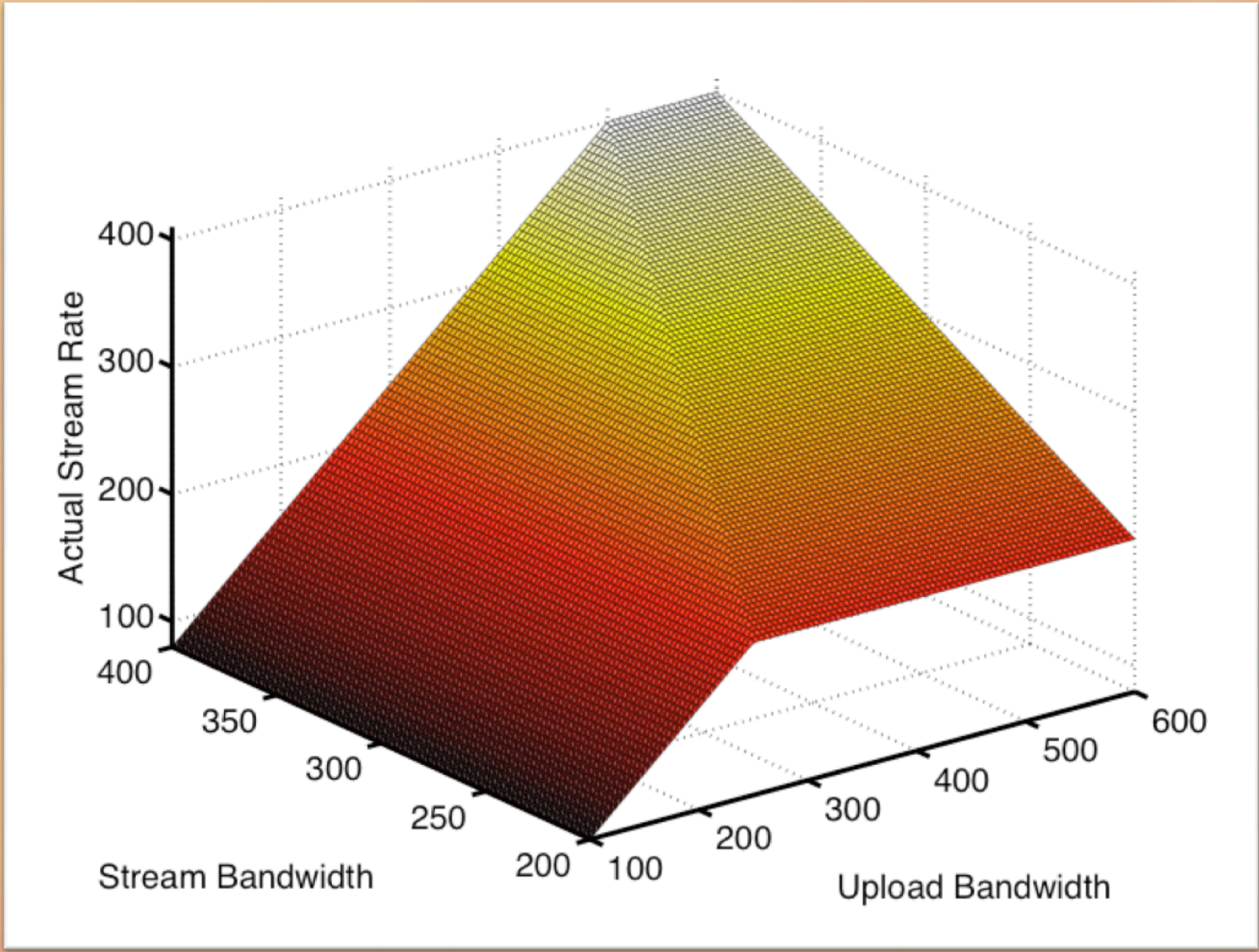
Simulations: Setup (Expected Returns)



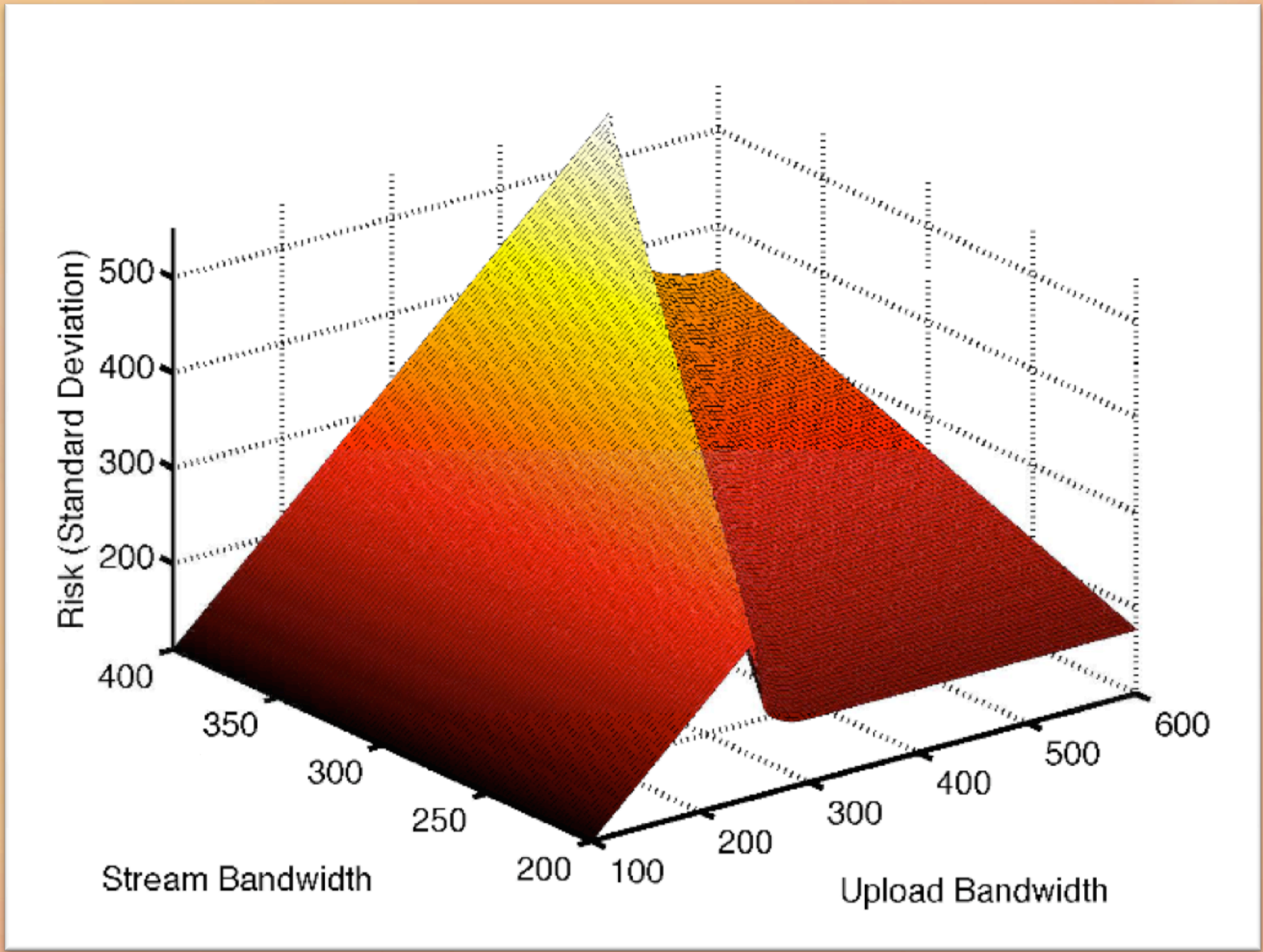
Simulations: Setup (Covariance Matrix)



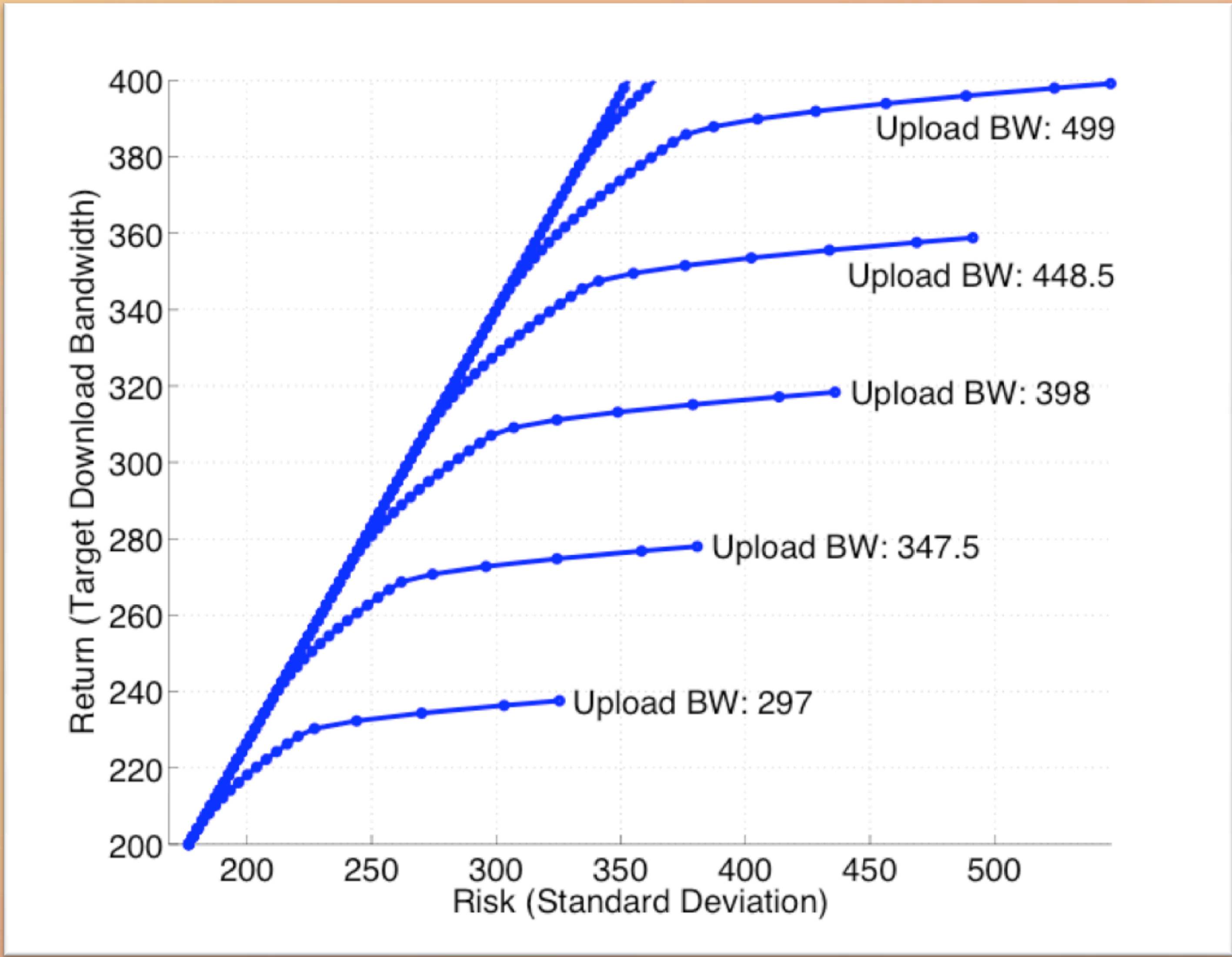
Simulations: Achievable Stream Rate



Simulations: Risk (Standard Deviation)



Simulations: Protocol Operation Curves



Conclusions

- A possible model for reciprocity-based peer-to-peer networks can be formulated based on **portfolio optimisation**
- The model can be extended:
  - Multi-stage formulations
  - Asymmetric risk measures
  - More general reciprocity models
    - See [Steinbach, 2001] and references therein
- Practical issues:
  - How can we measure the covariance matrix?

Thank You!

- Any Questions?

References

- Markowitz, H. M. (1952) “Portfolio Selection”. The Journal of Finance 7 (1): 77–91
- Markowitz, H.M. (1959) “Portfolio Selection: Efficient Diversification of Investments”. John Wiley & Sons.
- Steinbach, M. C. (2001) “Markowitz Revisited: Mean-Variance Models in Financial Portfolio Analysis”. SIAM Rev. 43 (1): 31-85