An Algorithm for Distributed Resource Allocation in QoS Overlays based on Vickrey-like Auctions

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Our Objective

• Auctions are a well-known way of performing distributed resource allocation in networking

• However, they can suffer from slow convergence

• We address this by proposing algorithms to
  – **Resolve auctions faster**, by auctioning many items at the same time
  – **Reduce the number of wasted bids**, by allowing peers to estimate the probability of a bid being successful
Sealed-envelope, Highest-losing bid Auction (SHA)

- An auctioneer \(i\) sells \(N_i\) indistinguishable items.

- Each bidder \(j\) sends a set \((b_{m_{ji}}, b_{m_{ji}-1}, \ldots, b_2, b_1)\) of bids \(b_{ij}\) for all the items it is interested in, each for a value \(v_{ij}\).

- Auctioneers rank all the received bids in increasing order, and the top \(N_i\) win the items.

- Each winning peer pays the value of the highest losing bid for its item.
Revelation Properties of the SHA

- We have proved analytically that truthful revelation of value is a dominant strategy equilibrium:
  - Conjecture, for each bidder, a bidding strategy
    \[
    \left( b_{m_ji}^{ji}(v_{ij}), b_{m_ji-1}^{ji}(v_{ij}), \ldots, b_2^{ji}(v_{ij}), b_1^{ji}(v_{ij}) \right)
    \]
  - Formulate peer utility as a function of this strategy
    \[
    \phi_j = \int_0^{b_{1}^{ji}} m_{ji}(v_{ij} - y)f_V(y)dy + \sum_{k=1}^{k=m_{ji}-1} \int_{b_{k}^{ji}}^{b_{k+1}^{ji}} (m_{ji} - k)(v_{ij} - y)f_V(y)dy
    \]
  - Find the strategy that maximizes peer utility
    \[
    b_{m_{ji}}^{ji} = b_{m_{ji}-1}^{ji} = \ldots = b_2^{ji} = b_1^{ji} = v_{ij}
    \]
Estimating SHA Outcomes

- Use of statistics of the \( k \)-th smallest value of a multi-item statistical sample (order statistics)
- Express \( P(V^i(k) \leq v_{ij}) \) in terms of order statistics

\[
P(V^i(k) \leq v_{ij}) = \int_0^{v_{ij}} \frac{m_{ji}!}{(k-1)!(m_{ji}-k)!} F_{V^i}(\omega)^{k-1}(1 - F_{V^i}(\omega))^{m_{ji}-k} f_{V^i}(\omega) d\omega
\]

- Find the ranks of the order statistics for which a peer wins a given number of auctions

\[
k_0 = \max(1, M_i - N_i + 1)
\]
\[
k_1 = \min(k_0 + m_{ji} - 1, M_i)
\]
Estimating SHA Outcomes

• For instance, assume that there are 6 items for sale ($N_i = 6$). Then:

$$k_0 = M_i - 5$$

<table>
<thead>
<tr>
<th>Bid Rank</th>
<th>$M_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_i - 1$</td>
</tr>
<tr>
<td></td>
<td>$M_i - 2$</td>
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<tr>
<td></td>
<td>$M_i - 3$</td>
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<td></td>
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<td>$\vdots$</td>
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Estimating SHA Outcomes

• For instance, assume that there are 6 items for sale \( N_i = 6 \). Then:
  \[ k_0 = M_i - 5 \]

• \( k_1 \) will depend on how many bids the peer sends.

<table>
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<tr>
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<tr>
<td>( M_i )</td>
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</tr>
<tr>
<td>( M_i - 1 )</td>
<td>5</td>
</tr>
<tr>
<td>( M_i - 2 )</td>
<td>4</td>
</tr>
<tr>
<td>( M_i - 3 )</td>
<td>3</td>
</tr>
<tr>
<td>( M_i - 4 )</td>
<td>2</td>
</tr>
<tr>
<td>( M_i - 5 )</td>
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If the value of bid of the peer is ranked here, it can win at most 1 item - independently of how many bids it sends.
Estimating SHA Outcomes

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If the value of bid of the peer is ranked here, it can win at 1 item if it sends 1 bid, and 2 items if it sends more.

- For instance, assume that there are 6 items for sale ($N_i = 6$). Then:
  \[ k_0 = M_i - 5 \]

- $k_1$ will depend on how many bids the peer sends.
Estimating SHA Outcomes

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If the value of bid of the peer is ranked here, it can win at 1 item if it sends 1 bid, and 2 items if it sends 2, or 3 items if it sends more.
Estimating SHA Outcomes

- We can now calculate the expected number of items won, given that $N_i$ are bid for at a value of $v_{ij}$.

$$\mu(v_{ij}, N_i, m_{ji}) = \sum_{k = k_0(v_{ij}, N_i)}^{k_1(v_{ij}, N_i)} I(F_{V_i}(v_{ij}); k, m_{ji} - k + 1)$$

- Where $I(x; k, n)$ is the regularized, incomplete beta function.

- The standard deviation can be found equivalently.
Simulation

• To test the analytic results presented before, we use an artificial bimodal value distribution as shown.

• Bids are generated according to this distribution, and the expected number of items won is recorded for discrete values of $v_{ji}$.
Estimating the number of bids won

- **Offered Items: 1**
- **Offered Items: 5**
- **Offered Items: 10**
- **Offered Items: 15**
- **Offered Items: 20**
- **Offered Items: 25**
- **Offered Items: 30**
- **Offered Items: 35**
- **Offered Items: 40**

**Analytically predicted outcome**

**Results from simulation**

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An Algorithm for Distributed Resource Allocation in QoS Overlays based on Generalised Vickrey Auctions

MSN 2009

Friday, 10 July 2009
Estimating the variability in bids won

Offered Items: 1

Offered Items: 5

Offered Items: 10

Offered Items: 15

Offered Items: 20

Offered Items: 25

Offered Items: 30

Offered Items: 35

Offered Items: 40

Analytically predicted outcome

Results from simulation
SHA and P2P Live Streaming Overlays

• We use auctions to perform capacity/delay tradeoffs in P2P streaming overlays.

• For each chunk they are interested in,
  – Peers send a bid to the peer that gives them the greatest value for that chunk;
  – If the expected number of times the chunk will be received is smaller than 1, the process is repeated.
  – The process takes peer capacity, delay and load into account.
De-centralised Overlay Topology Construction

An Algorithm for Distributed Resource Allocation in QoS Overlays based on Generalised Vickrey Auctions
System converges to a “tree-like” equilibrium topology, with peers with lower delays from the source uploading much more actively than peers with larger delays. This tends to minimize stream delay.
De-centralised Overlay Topology Construction

Good stream delay from the source, only around 40/50% higher than for a direct connection from the stream originating peer.
Future Work: Lost Chunks ("Slips")

Effect related to the variance endogenous to the bidding method.

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Future Work: High Chunk Jitter
Thank You!

Questions?
High Chunk Delay

Suboptimal bidding due to interactions between chunk search and bidding