Power Domain Representations of Spaces of Compact Subsets

Petter Kristian Køber

Department of Mathematics, University of Oslo

A *domain representation* of a topological space X consists of a domain D, a subset $D^R \subseteq D$ and a continuous and surjective function $\varrho : D^R \longrightarrow X$. It is *countably based* if D is.

The Plotkin power domain construction is a useful tool for representing non-determinism. We look at the possibility of representing some or all non-empty compact subsets of a topological space X over the Plotkin power domain $\mathbb{P}_P(D)$, using a certain standard domain representation of X over D. In the case of a complete metric space, this has already been investigated in [2]: The represented space consists of all non-empty compact subsets of X, and the induced quotient topology coincides with the one generated by the Hausdorff metric.

The T_0 quotients of countably based spaces (qcb_0 spaces) have been characterised as those topological spaces which can be given admissible and countably based standard representations, and we restrict our study to these spaces. One possible characterisation of the Plotkin power domain $\mathbb{P}_P(D)$ is as a certain free domain algebra over D, and a similar construction exists for qcb_0 spaces [1]. We look at the relation between these two constructions and also compare the latter to what might be obtained using a standard domain representation.

The author [4] has previously demonstrated how a qcb_0 space can be defined via a strictly positive induction, e.g. as the least fixed point of an equation $X = A \uplus [B \Rightarrow X]$, where A and B are parameters, $\uplus$ the disjoint union and $\Rightarrow$ the exponential in the category of qcb_0 spaces and continuous functions. We discuss the possibility of extending these definitions using representations of spaces of compact subsets over the Plotkin power domain.

References

1. I.Battenfeld, Computational Effects in Topological Domain Theory. Electronic Notes in Theoretical Computer Science **158**, 59-80, Elsevier 2006.
2. J.Blanck, Effective Domain Representations of $\mathcal{H}(X)$, The Space of Compact Subsets. Theoretical Computer Science **219**, 19-48, Elsevier 1999.
3. G.Hamrin, Admissible Domain Representations of Topological Spaces. U.U.D.M.Report 16, Uppsala University 2005.
4. P.Køber, Qcb_0 spaces defined by strictly positive induction. Manuscript 2008.