From Parametric Polymorphism to Models of Polymorphic FPC

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Dual intuitionistic / linear lambda calculus (DILL) is a type theory in which terms have two variable contexts: an intuitionistic context and a linear context. DILL can be modeled in domain theory by interpreting types as pointed complete partial orders, and modeling intuitionistic and linear variable dependence using respectively continuous and strict continuous functions. The ability to express strictness in the type theory is useful for reasoning about recursion as this is needed to express for example the uniformity principle for fixed points in domain theory: if $hf = gh$ then $\mathit{fix}\, g = h(\mathit{fix}\, f)$ holds if h is strict. Plotkin noticed that DILL was the right type theory for reasoning about the combination of recursion and parametric polymorphism, and suggested moreover that the type theory obtained from adding parametric polymorphism and fixed points to DILL (this type theory is henceforth referred to as PILL_Y) could be used as an axiomatic setup for domain theory because general recursive types can be encoded in PILL_Y as polymorphic types.

In this talk we test this thesis by showing how every model of parametric PILL_Y gives rise to a model of FPC, a simply typed lambda calculus with recursive types and a call-by-value operational semantics, mimicking a classical result from domain theory in the setting of parametric PILL_Y. In fact, we prove that more generally any algebraically compact linear category gives rise to a model of FPC. That this can be done is nontrivial essentially because the recursive types live in the linear part of PILL_Y and FPC is not a linear calculus. The solution presented here uses Girard's lesser known second translation of intuitionistic type theory into linear type theory, extending it to recursive types. Surprisingly, this translation does not seem to extend to a polymorphic version of FPC even though parametric PILL_Y has polymorphic types.

If time permits I will also sketch the FPC model obtained by applying the general theory to a PILL_Y model based on admissible pers (partial equivalence relations) over a reflexive domain. This particular model is interesting for two reasons: (1) we can extend the interpretation to polymorphic FPC, and (2) the interpretation is computationally adequate.

The results presented in talk appeared first in an ICALP 2006 paper.