Basing Markov transition systems on the Giry monad

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M. Giry introduced in 1981 a monad that addresses the categorical foundation of the measure theoretic part of probability theory; this monad —actually a family of monads — is usually called the *Giry monad* in her honor. It permits stating many properties of stochastic relations (or of *Markov kernels*, as they are called in probability theory) in terms of a monad, so that its rôle is comparable to that of the power set monad for set-theoretic relations. We plan to discuss some developments for stochastic coalgebras, probabilistic interpretations of modal and coalgebraic logic and weak morphisms of Markov transition systems through the Giry monad. Particular attention will be given to issues pertaining to the theory of Borel sets, viz., basing the monad on the category of Polish resp. analytic spaces, which may change the monad's property in subtle ways. We want to indicate that the Giry monad yields in fact a uitable framework for the theory of stochastic relations and of Markov transition systems, sometimes in parallel to the power-set monad, sometimes with strikingly different properties.