# Towards a Logic of Sequential Computation:

**A synthetic account of Sequential Domain Theory**

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Synthetic Domain Theory (SDT) was originally suggested by Dana Scott to obtain a uniform and logic-based account of domain theory. In SDT the domain structure is intrinsic to a chosen class of sets with "good" properties.

SDT is uniform in the sense that it applies to different models of the basic axioms thus giving rise to different kinds of domains. E.g. in [2] one finds a sheaf model for a kind of "stable" SDT and in [6, 3] realizability models for a kind of "strongly stable" domain theory in the sense of [1]. Axiomatising these different kinds of models would give rise to different kinds of "flavours" of SDT as suggested by Martin Hyland.

The pca used in [6, 3] is derived from the universal object $U = [N \to N]$ of the category $\mathcal{SA}$ of countably based sequential algorithms (where $N$ is the concrete data structure of natural numbers). As shown in [4, 5] the type $U = [N \to N]$ is also universal in the wellpointed category $\mathcal{OSA}$ of *observably sequential algorithms* which contains $\mathcal{SA}$ as a lluf sub-ccc. In this paper we consider the realizability model for SDT arising from $U$ in $\mathcal{OSA}$ and discuss possible axiomatizations of sequential SDT inspired by this model. In particular, we exploit the fact that there is a type $O$ such that $O \to O$ contains a dominance $\Sigma$ corresponding to the one considered in the realizability model over $U$ in $\mathcal{SA}$.

## References

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