First-order definability of LRT

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In the first author's thesis [2], and further publications [3, 4] a sequential language, LRT, for real number computation is investigated. The thesis includes a proof that all polynomials are programmable, but that work comes short of giving a complete characterization of the expressive power of the language even for first-order functions. The technical problem is that LRT is non-deterministic. So a natural characterization of its expressive power should be in terms of relations rather than functions. In [1], Brattka investigates a formalization of recursive relations in the style of Kleene's recursive functions on the natural numbers. This paper establishes the expressive power of LRT_p, a variant of LRT, in terms of Brattka's recursive relations. Because Brattka already did the work of establishing the precise connection between his recursive relations and Type 2 Theory of Effectivity, we thus obtain a complete characterization of first-order definability in LRT_p. The corresponding denotational semantics employs several ideas familiar to domain theorists, including measurement as defined by Martin in [5] and a monadic treatment of the distinction between value and computation as in Moggi [6]. Furthermore, because we are interested in relations, say, on product types, we also extend the language to have explicit products of ground types.

Referencias

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