Abstract: Sequential Algorithms for Unbounded Nondeterminism

J. Laird
Department of Computer Science, University of Bath

Unbounded nondeterministic choice is computationally interesting, not least because it is co-expressive with *fair* choice. Giving an informative denotational semantics for functional programs with unbounded choice is challenging because they are not, in general continuous with respect to the Scott ordering. In previous work we have shown that by interpreting programs as stable, but non-continuous functions in Berry-style biorders with infinite meets we may give a fully abstract cps interpretation of countable nondeterminism.

However, this account does not explicitly capture the *behaviour* of programs with unbounded choice, and indeed there has been no "intensional semantics" with respect to total correctness which does so. Again, non-continuity raises some interesting problems: to establish termination of a program in a given environment typically requires an infinitary derivation, and so such a model should characterize the ordinal bounds on these evaluations.

In this work, we give an intensional description of stable, monotone functions as nondeterministic sequential algorithms on ordered concrete data structures. (Deterministic) sequential algorithms on (unordered) concrete data structures have been shown to be in bijective correspondence with *bistable* and continuous functions on associated bistable biorders. Here, we extend these results to a setting with unbounded nondeterminism, defining concrete data structures in which the sets of cells, values and events are ordered, and in which proofs of events (representing interactions between programs and their environments) are ordinal sequences. The upper states over an ordered CDS form a biorder, and we show that the stable and monotone functions between these biorders are in bijective correspondence with the sequential algorithms (states of the function-space OCDS). Hence ordered concrete data structures and nondeterministic sequential algorithms form a Cartesian closed category.

Working with sequential algorithms as intensional representations of stable and monotone functions yields a rather more illuminating proof of full abstraction for our CPS language with countable choice. This has a simple "universal" type, and we shall describe the algorithms inhabiting it. We shall also discuss the solution of domain equations in this setting.